Why black holes with permeable event horizons are perpetual motion machines

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A perpetual motion machine is a hypothetical machine that violates the conservation of energy by producing more energy than it consumes. Because of the violation of the conservation of energy, perpetual motion machines exist only hypothetically, not in physical reality. Here is shown that penetration of the event horizon of a black hole results in a violation of the conservation of energy possible only in a hypothetical perpetual motion machine.

The energy equivalence E_M of a system consisting only of mass M is well known to be

$$E_M = Mc^2 . \quad (1)$$

[1]

If a mass m is added to the system, the equivalent energy E added to the system as a result of the presence of mass m is also well known to be

$$E = mc^2 . \quad (2)$$

Thus if the system now consisting of mass M and mass m were dissolved into radiation, the total energy E_T of the resulting radiation would be equal to

$$E_T = E_M + E = Mc^2 + mc^2 . \quad (3)$$

Since the total energy equivalence added to the system by the addition of mass m is $E = mc^2$, any gravitational energy or kinetic energy added to the system as a result of the presence of mass m must be included as part of the additional energy E described in equation (2).

If mass M is a black hole and mass m is a particle, values for components of energy can be obtained from the Schwarzschild metric,[2]

$$c^2 d\tau^2 = c^2(1-\frac{R}{r})dt^2 - \frac{dr^2}{(1-R/r)} - r^2 d\theta^2 - \left(r^2 \sin^2\theta\right)d\varphi^2. \qquad (4)$$

In equation (4), r, θ and φ are Schwarzschild space coordinates indicating the location of mass m relative to the location of the black hole, t is the Schwarzschild time coordinate, τ is the proper time coordinate for mass m, c represents the speed of light in a vacuum and R is the Schwarzschild radius that is located at the event horizon of the black hole.

Equation (4) can be rewritten as

$c^2\left(\frac{d\tau}{dt}\right)^2 = c^2(1-\frac{R}{r})\left(\frac{dt}{dt}\right)^2 - \frac{1}{(1-R/r)}\left(\frac{dr}{dt}\right)^2 - r^2\left(\frac{d\theta}{dt}\right)^2 - \left(r^2\sin^2\theta\right)\left(\frac{d\varphi}{dt}\right)^2$, which can be rearranged as $c^2 = c^2\left(\frac{d\tau}{dt}\right)^2 + c^2\frac{R}{r} + \frac{1}{(1-R/r)}\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 + \left(r^2\sin^2\theta\right)\left(\frac{d\varphi}{dt}\right)^2$, which can also be expressed as

$$E = mc^2 = mc^2\left(\frac{d\tau}{dt}\right)^2 + mc^2\frac{R}{r} + m\frac{1}{(1-R/r)}\left(\frac{dr}{dt}\right)^2 + mr^2\left(\frac{d\theta}{dt}\right)^2 + m\left(r^2\sin^2\theta\right)\left(\frac{d\varphi}{dt}\right)^2. \qquad (5)$$

In order to increase intuitive understanding of equation (5) it is helpful to group the terms set out in equation (5) into components. The grouping is accomplished by defining a variable v where $v^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 + \left(r^2\sin^2\theta\right)\left(\frac{d\varphi}{dt}\right)^2$. Variable v represents the portion of the total velocity of mass m that is purely dependent upon motion through space and independent of gravity. Since $\frac{1}{(1-R/r)} = \frac{R}{(r-R)} + 1$, equation (5) can be rearranged as

$$E = mc^2 = mc^2\left(\frac{d\tau}{dt}\right)^2 + mc^2\frac{R}{r} + m\frac{R}{(r-R)}\left(\frac{dr}{dt}\right)^2 + m\left(\frac{dr}{dt}\right)^2 + mr^2\left(\frac{d\theta}{dt}\right)^2 + m\left(r^2\sin^2\theta\right)\left(\frac{d\varphi}{dt}\right)^2.$$

Replacing $\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 + \left(r^2\sin^2\theta\right)\left(\frac{d\varphi}{dt}\right)^2$ with v^2 yields

$$E = mc^2 = mc^2\left(\frac{d\tau}{dt}\right)^2 + mc^2\frac{R}{r} + m\frac{R}{(r-R)}\left(\frac{dr}{dt}\right)^2 + mv^2. \qquad (6)$$

Equation (6) shows the energy $E = mc^2$ being apportioned into four energy components. The energy component mv^2 is a motion component of energy that is dependent solely on velocity of mass *m* through space.

The energy component $mc^2\frac{R}{r}$ is a gravitational component of energy that is inversely proportional to the radial distance of mass *m* from the center of the black hole.

The energy component $m\frac{R}{(r-R)}\left(\frac{dr}{dt}\right)^2$ is a hybrid component that is dependent both on the radial distance of mass *m* from the center of the black hole and velocity of mass *m* in a radial direction.

The final energy component $mc^2\left(\frac{d\tau}{dt}\right)^2$ in equation (6) is based on the different rates at which time passes when measured by the Schwarzschild time coordinate *t* and when measured by the proper time coordinate τ of mass *m*.

In equation (6), the energy $E = mc^2$ represents the amount of energy "consumed" by the addition of mass *m* to the system consisting of the black hole to form a new system consisting of mass *m* and the black hole. The four components of energy represent how the energy is manifested within the new system, i.e., the four components of energy represent the amount of energy "produced" by the addition of mass *m* to the system consisting of the black hole. As long as the sum of the four components is equal to mc^2, energy is conserved. However, when the sum of the four components exceeds mc^2, more energy is "produced" than is "consumed" by the formation of the new system, making the new system a perpetual motion machine.

When mass *m* is at a radial location outside the event horizon (i.e., $r>R$), energy component $mc^2 \frac{R}{r} < mc^2$ providing no barrier to the sum of the four components being equal to mc^2. When mass *m* is at a radial location inside the event horizon (i.e., $r<R$), energy component $mc^2 \frac{R}{r} > mc^2$. When mass *m* is at $r=0$, energy component $mc^2 \frac{R}{r} = \infty$.

For mass *m* to reach and cross the event horizon would require the energy component $mc^2 \frac{R}{r}$ to reach and exceed $E = mc^2$ making the sum of the four energy components greater than the available energy (i.e., $E = mc^2$). That is, the energy produced by the new system composed of the black hole and mass *m* located within the event horizon is greater than the energy consumed to form the new system (i.e., $E = mc^2$), making the new system a perpetual motion machine. For mass *m* to reach $r=0$ requires the sum of the energy components to be infinite, making the new system a perpetual motion machine par excellence, consuming finite energy while producing infinite energy.

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[1] A. Einstein, "On the Electrodynamics of Moving Bodies," translated from "Ist die Trägheit eines Körpers von seinem Energiegehalt abhängig?" Annalen der Physik, 17, 1905, in *The Principle of Relativity*, Dover Publications, Inc. 1952, pp. 69-71.
[2] K. Schwarzschild, Sitzungsberichte der Koeniglich Preusichen Akademie der Wissenschaften zu Berlin, Phys.-Math. Klasse, 189 (1916), translated by S. Antoci, A Loinger, arXiv:Physics/9905030v1 (1999).