The fiery end of a journey to the event horizon of a black hole

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Light transmitted towards the event horizon of a black hole will never complete the journey. Either the black hole will disintegrate or the light itself will disintegrate before the light can reach the event horizon. The incomplete journey illustrates how locations where time is dilated observe and experience an increase in the rate that things disintegrate. When a mass is compacted so that the Schwarzschild radius is near its surface, the very significant increase in time dilation at the surface results in a corresponding increase in the rate of surface disintegration, explaining the existence of quasars.

The progress of light transmitted radially toward the event horizon, located at the Schwarzschild radius *R*, of a black hole can be monitored with the use of measuring stations located between a light transmitter and the event horizon.

The distance and the time for light to travel from the light transmitter to each measuring station can be calculated from the Schwarzschild metric[1],

$$c^2 d\tau^2 = c^2(1-\frac{R}{r})dt^2 - \frac{dr^2}{(1-R/r)} - r^2 d\theta^2 - \left(r^2 \sin^2\theta\right)d\varphi^2 . \tag{1}$$

The first step in the calculation is to determine values for time dilation and length contraction for each measuring station located a radial distance *r* from the center of the black hole. Each measuring station uses the proper time coordinate τ to measure proper travel time and the proper radial coordinate ρ to measure proper radial distance. The Schwarzschild time coordinate *t* is used to measure coordinate time.

The light transmitter and the measuring stations are defined to be stationary with respect to the Schwarzschild space coordinates $(r,\ \theta, \varphi)$ so that $dr = d\theta = d\varphi = 0$. This reduces the Schwarzschild metric to

$$c^2 d\tau^2 = c^2(1-\frac{R}{r})dt^2. \tag{2}$$

Solving equation (2) for $d\tau$ yields

$$d\tau = dt\sqrt{1-\frac{R}{r}}. \tag{3}$$

In order to preserve general relativity it is necessary that at every location length contraction is equal to the inverse of time dilation;[2] therefore, $\frac{dr}{d\rho} = \frac{d\tau}{dt}$, allowing equation (3) to be rewritten as

$$d\rho = \frac{dr}{\sqrt{1-R/r}}. \tag{4}$$

Equation (3) and equation (4) can be used to obtain the distance from the light transmitter to a measuring station located at any radial location $r \geq R$, as measured using proper radial coordinate ρ as well as the travel time measured using either time coordinate t or proper time coordinate τ.

Equation (4) provides the integrand, $\frac{dr}{\sqrt{1-R/r}}$, used to calculate the proper radial distance $\Delta\rho$ between the light transmitter located at radial location r_L and a measuring station located at radial location r_M, i.e.,

$$\Delta\rho = \int_{r_M}^{r_L} \frac{dr}{\sqrt{1-R/r}}. \tag{5}$$

The speed of light is the same in every reference frame, that is $\frac{d\rho}{d\tau} = c$ and therefore, $d\rho = cd\tau$. Substituting this value into equation (4) yields $d\tau = \frac{dr}{c\sqrt{1 - R/r}}$ providing the integrand $\frac{dr}{c\sqrt{1 - R/r}}$ used to calculate the proper time interval $\Delta\tau$ (i.e., proper travel time) for light to travel from the light transmitter located at radial location r_L to the measuring station located at radial location r_M, i.e.,

$$\Delta\tau = \int_{r_M}^{r_L} \frac{dr}{c\sqrt{1 - R/r}}. \tag{6}$$

From equation (3) and equation (4), $\frac{d\rho}{d\tau} = \frac{dr}{dt}\frac{1}{1 - R/r}$. Recognizing that for light $\frac{d\rho}{d\tau} = c$ yields $dt = \frac{dr}{c(1 - \frac{R}{r})}$, providing the integrand $\frac{dr}{c(1 - \frac{R}{r})}$ used to calculate the time interval Δt (i.e., coordinate travel time) for light to travel from the light transmitter located at radial location r_L to the measuring station located at radial location r_M, i.e.,

$$\Delta t = \int_{r_M}^{r_L} \frac{dr}{c\left(1 - R/r\right)}. \tag{7}$$

The distance and travel time from the light transmitter to the event horizon can be calculated by treating the Schwarzschild radius *R* as the location of a measuring station.

The proper distance $\Delta\rho$ from the light transmitter to the event horizon is calculated by performing the integral in equation (5) with the limits *R* and r_L. The integral is convergent indicating proper distance $\Delta\rho$ is finite.

The proper time interval $\Delta\tau$ is calculated by performing the integral in equation (6) with the limits R and r_L. The integral is convergent indicating proper time interval $\Delta\tau$ is finite.

The time interval Δt is calculated by performing the integral in equation (7) with the limits R and r_L. The integral is divergent indicating time interval Δt is infinite.

When measured by coordinate time *t*, the journey of light to the event horizon is not completed in finite time. Nevertheless, using equations (7), (5) and (6), every finite value of time *t* that occurs on light's journey from the light transmitter to the event horizon can be mapped, respectively, to a specific value of radial distance *r*, a specific value of proper radial distance ρ, and a specific value of proper time τ.

It is generally accepted that black holes are not eternal, but will disintegrate within approximately 10^{55} years.[3] Assuming that light from the light transmitter does not disintegrate first, a radial location r_D reached by the light in 10^{55} years can be calculated using equation (7). The calculations shows r_D>R.

The proper distance $\Delta\rho$ travelled by the light before the black hole disintegrates is calculated performing the integral in equation (5) with the limits r_D and r_L. The proper distance $\Delta\rho$ calculated with these limits is less than the proper distance to the event horizon.

The proper time interval $\Delta\tau$ is calculated performing the integral in equation (6) with the limits r_D and r_L. The proper time interval $\Delta\tau$ calculated with these limits is less than the proper time interval required for light to reach the event horizon.

As shown above, light emitted from the transmitter will not reach the event horizon of a black hole, in coordinate time or in proper time, before the black hole disintegrates.

While in the above example it was assumed that light is able to propagate for 10^{55} years, it is possible that the light itself will disintegrate before the black hole disintegrates. For this case, the radial location r_D where the light disintegrates can be used in equations (5) and (6) to, respectively, calculate the corresponding proper distance $\Delta\rho$ and proper time interval $\Delta\tau$ to the location of light disintegration. The calculations confirm that any light disintegration will occur before the light reaches the event horizon.

The journey of a particle from the location of the light transmitter to the event horizon of the black hole is similar to the journey of light as described above, except however, the particle's journey will be shorter in both distance and time. This is because the particle will travel at a slower velocity. The particle will also be less hardy than light and therefore will disintegrate before the black hole disintegrates at a measuring station between the light transmitter and the event horizon. The radial location r_D of the measuring station where the particle disintegrates can be used in equations (6) and (5) to calculate the corresponding proper distance $\Delta\rho$ and proper time interval $\Delta\tau$ for light to reach the measuring station where the particle disintegrates before reaching the event horizon.

In essence, the particle experiences an increase in its rate of disintegration as it approaches the event horizon of a black hole. This increase in disintegration rate is a logically necessary result of general relativity. Increasing entropy, as observed from a low gravity field, occurs everywhere as the universe unwinds. The same increase in entropy occurs at a faster rate when observed or experienced in a higher gravity field because of the dilation of time there. As the particle approaches the event horizon of the black hole, time dilation approaches infinity, which accelerates the increase of entropy accordingly. As long as the particle (or if the particle were very hardy, the black hole) disintegrates in finite coordinate time, this will be experienced by the particle as an increase in the rate of its own disintegration (or that of the black hole) preventing the particle from reaching the event horizon.

The very significant increase in the rate of disintegration that is observed and experienced near the event horizon of a black hole, and by logical extension in space near the Schwarzschild radius of any mass, makes for a very intriguing picture of the collapse of a star, explaining the existence of quasars. Consider, for example, the following potential scenario of the collapse of a supergiant, from the perspective of the surface, as the star compacts and the surface makes a run at the Schwarzschild radius. At the start, tremendous mass and acceleration produces a seemingly unstoppable downward momentum.

As the surface of the collapsing star approaches the Schwarzschild radius, the rate of disintegration increases noticeably. Before reaching the Schwarzschild radius, surface layers begin to radiate into space. The closer to the Schwarzschild radius the surface approaches, the faster the rate of disintegration. If the surface of the collapsing star could draw very near the Schwarzschild radius, the rate of disintegration would be almost infinite.

The loss of mass at the surface of the collapsing star will hinder the surface's attempt to overtake the Schwarzschild radius. The lost of mass at the surface will reduce the volume V of the star, correspondingly decreasing the radius r in accordance with the well known relationship

$$r = \sqrt[3]{V\frac{3}{4\pi}}\,. \tag{8}$$

The loss of mass will also decrease the value of the Schwarzschild radius R in accordance with the well known relationship

$$R = \frac{2GM}{c^2}, \tag{9}$$

where M represents the mass of the star and G represents the gravitational constant.

Because of varying density, mass M may only roughly correspond to volume V; nevertheless, equation (8) and equation (9) clearly indicate that removal of mass at the surface of the collapsing star will reduce R at a significantly faster rate than it will reduce r. Therefore, the loss of mass at the surface will result in an increase in the distance between r and R pushing the Schwarzschild radius back down away from the surface. The collapsing surface may continue for a time charging down toward the Schwarzschild radius, but it will never catch it. When the downward momentum of the collapsing star is spent, the Schwarzschild radius will remain below the surface. During the collapse, however, a large amount of mass, perhaps many times the size of our own sun, will have been almost instantaneously turned to radiation.

From a distance, this inferno of a proportion rarely matched in the universe will at first be, at least partially, masked by the dilation of time. However, when the collapse slows or pauses allowing the Schwarzschild radius to descend back down away from the surface, a faster playing clip of the remnants of the blaze may be released, that even in its diminished form can dazzle.

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[1] K. Schwarzschild, Sitzungsberichte der Koeniglich Preusichen Akademie der Wissenschaften zu Berlin, Phys.-Math. Klasse, 189 (1916), translated by S. Antoci, A Loinger, arXiv:Physics/9905030v1 (1999).
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[3] S. W. Hawking, Nature (London) 248, 30 (1974).